Number Theory

9-12 Grade

Science Based Math

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Executive Summary

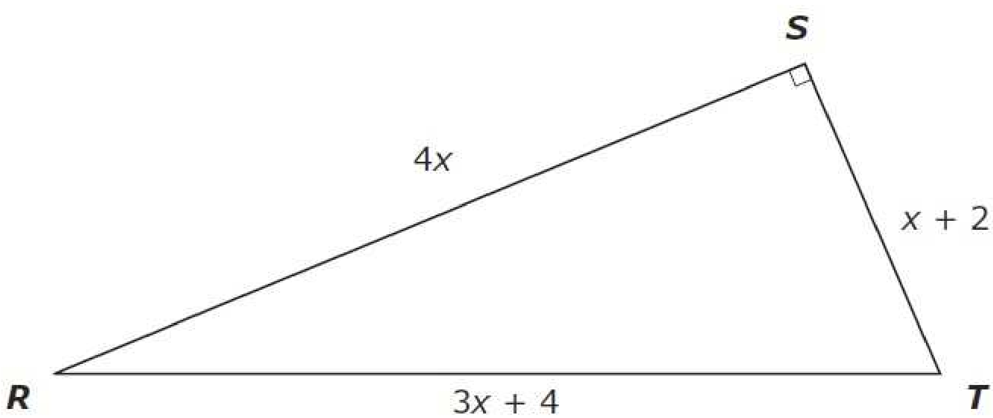
Unit One: Vectors Arithmetic
Days Required: 5
Lesson Objective: Adding and subtracting vectors and finding the resultant, intuitively done before being formalized.

Unit Two: Accuracy/Precision and Significant Figures
Days Required: 8
Lesson Objective: This unit provides an understanding of accuracy, precision and significant figures. The topic is usually encountered within the first chapter and first days of a general chemistry course. Understanding this is important particularly in regards to any laboratory experience. Although there are numerous explanations provided in textbooks, they often use overly abstract analogies. The analogies often leave the students as much in the dark just as much as the central problem did

Unit Three: Scientific Notation and Metric Prefixes
Days Required: 3
Lesson Objective: Students will be able to manipulate and interpret very large numbers and very small numbers expressed in the "exponent" format of scientific notation. Students will also be able to understand how to use metric prefixes (centi, milli, kilo, mega, etc) as a shorthand for commonly used scientific notations.

Sample MCA Question

Triangle *RST* is shown.



How many units long is $\overline{RS}$?

A. 2
B. 3
C. 4
D. 12

Table of Contents

Unit 1: Vector Arithmetic

Lesson Objective: Add vectors and find the resultant.

Days of Instruction: 5

Day 1: Pre-Test, Launch and Explore
Day 2: Continue the Explore activity and Summarize
Day 3: Have students work in small groups then share their results on the whiteboard using the problems in the problem set, selecting one problem from each section
Day 4:Have students work in small groups then share their results on the whiteboard using the problems in the problem set, selecting one problem from each section
Day 5: Finish clarifying any questions students have, have them work out three problems then finish up with the Post Test

Minnesota State Standard(s):

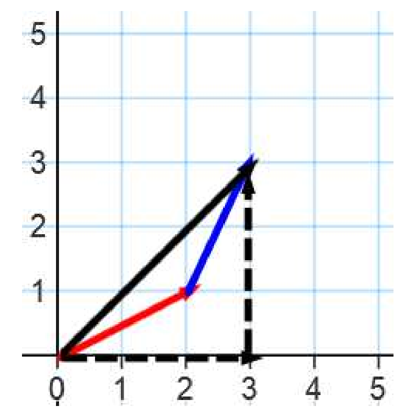
Math	Geometry & Measurement	Know and apply properties of geometric figures to solve real world and mathematical problems and to logically justify results in geometry.	9.3.3.4	Apply the Pythagorean Theorem and its converse to solve problems and logically justify results.
Math	Geometry & Measurement	Solve real-world and mathematical Geometric problems using algebraic methods.	9.3.4.2	Apply the trigonometric ratios sine, cosine and tangent to solve problems, such as determining lengths and areas in right triangles and in figures that can be decomposed into right triangles. Know how to use calculators, tables or other technology to evaluate trigonometric ratios.
Math	Geometry & Measurement	Know and apply properties of geometric figures to solve real world and mathematical problems and to logically justify results in geometry.	9.3.4.7	Use algebra to solve geometric problems unrelated to coordinate geometry, such as solving for an unknown length in a figure involving similar triangles, or using the Pythagorean Theorem to obtain a quadratic equation for a length in a geometric figure.
Physical Science	Motion	Forces and inertia determine the motion of objects.	9.P.2.2.1.1	Use vectors and free-body diagrams to describe force, position, velocity and acceleration of objects in two-dimensional space.

Launch: Introduce a question involving vector addition such as a boat traversing a river perpendicular to the flow of the river. Allow students time to debate how fast the boat would be traveling overall in different river currents and different boat speeds. They will not be able to give exact answers, but should begin to build intuitions about the fact that the resultant answer will be larger than the two individual speeds.

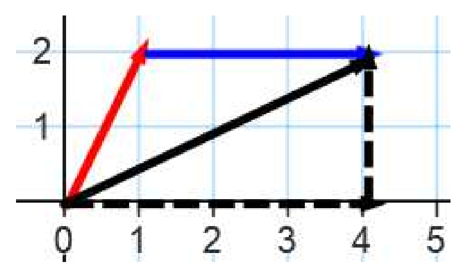
Explore: Students will use Geoboards to explore adding two vectors together in the <a, b> or ai + bj forms. Students will use a custom made “ruler” with 1 unit equalling the distance between any two adjacent pegs on the Geoboard to estimate the length of the resultant. Students will check their estimates against the exact length using the Pythagorean Theorem.

<u>Activity</u>

Show students how to represent a vector on a Geoboard, example of <2, 1> and <1, 2>. What is <2, 1> + <1, 2> and represent it on the Geoboard? What is <2, 1> + <1, 2> in < , >? What is the diagonal length of the resultant vector from the origin?



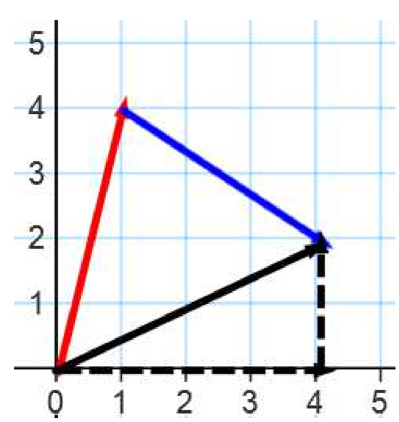
What is <2, 1> + <3, 0> and represent it on Geoboard? What is <2, 1> + <3, 0> in < , >? What is the diagonal length of the resultant vector?



Show <3, 0> + <2, 1>? How does that relate to the last question?

Is adding vectors commutative (can you add in any order)?

What is <1, 4> + <3, -2> and represent it on Geoboard? What is <1, 4> + <3, -2> in < , >? What is the diagonal length of the resultant vector?



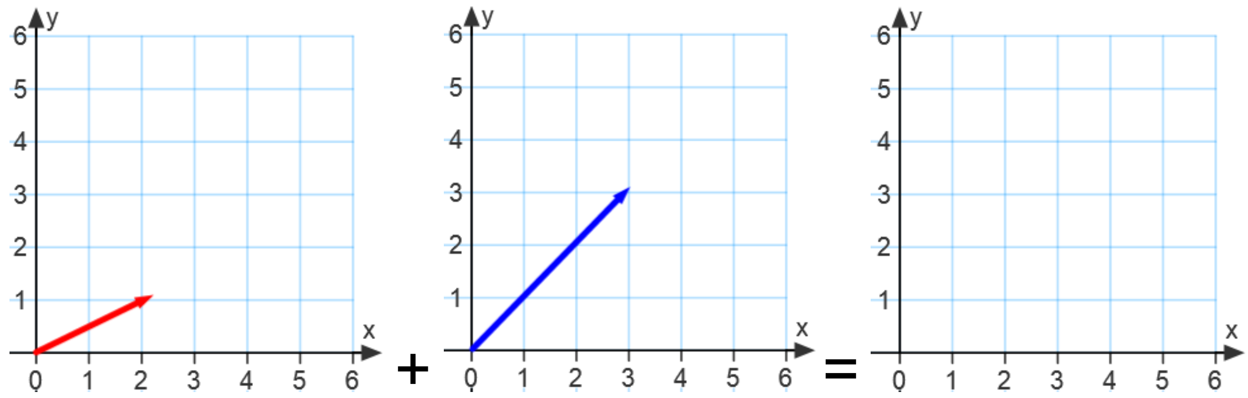
What is <a, b> + <c, d> in < , >? What is the diagonal length of the resultant vector?

Share: Students will share verbally in class and go to the whiteboard and share their findings and explain their work to the class.

Summarize: The teacher will summarize the main points of the lesson that $ai + bj + ci + dj = (a+c)i + (b + d)j$ and that to find the resultant you can use the Pythagorean Theorem and to find the direction (angle) of the resultant students can use the tangent function ($\tan\theta = y/x$)

Vectors Pre/Post Test

1. Use the space provided below to show the addition of the red and blue vectors.



2. Use $ai + bj + ci + dj = (a + c)i + (b + d)j$ to find the resultant of the two vectors shown above.

3. A river has a current of 3 m/s and is heading west. If you are on the south shore and swim straight north at 2 m/s, what speed and direction will you actually go?

4. If the river is 14 meters wide, how long will it take you to get to the other side?

5. How far will you have swam total by the time you get to the other side?

Problem set to choose problems for students to work on and share on the whiteboard.

Find a + b, a – b, 4a + 5b, 4a – 5b, and ‖a‖.

1. a= ⟨2,-3⟩ b= ⟨1,4⟩
2. a= -⟨7,-2⟩ b= 4⟨-2,1⟩
3. a= i+2j b=3i-5j

Sketch vectors corresponding to a, b, a + b, 2a, and -3b.

4. a = 3i + 2j b = -i + 5j
5. a= ⟨-4,6⟩ b= ⟨-2,3⟩

Find the magnitude of the vector a and the smallest positive angle θ from the positive x-axis to the OP that corresponds to a.

6. a= ⟨3,-3⟩
7. a= ⟨-5,0⟩
8. a = -4i + 5j
8. a = -18j

The vectors a and b represent two forces acting at the same point, and θ is the smallest positive angle between a and b. Approximate the magnitude of the resultant force. (hint: put one of the angles on the positive x-axis then draw the other angle)

9. ‖a‖=40 lb,‖b‖=70 lb,θ=45°
10. ‖a‖=2.0 lb,‖b‖=8.0 lb,θ=120°

The magnitudes and directions of two forces acting at a point P are given in (a) and (b). Approximate the magnitude and direction of the resultant vector.

11. a) 90 lb, N75°W b) 60 lb, S5°E
12. a) 6.0 lb, 110° b) 2.0 lb, 215°
13. Find a unit vector that has (a) the same direction as the vector a and (b) the opposite direction of the vector a. . a= -8i+15j
14. Find a vector that has the same direction as ⟨-6,3⟩ and
    a) twice the magnitude
    b) one-half the magnitude

Find (a) the dot product of the two vectors and (b) the angle between the two vectors.

16. ⟨-2,5⟩, ⟨3,6⟩
17. 4i – j, -3i + 2j
18. 9i, 5i + 4j
19. ⟨10,7⟩, ⟨-2,-7/5⟩

Show that the vectors are orthogonal.

20. ⟨4,-1⟩, ⟨2,8⟩
21. -4j, -7i

Show that the vectors are parallel, and determine whether they have the same direction or opposite directions.

22. a=3i-5j b= -12/7 i+20/7 j
23. a= ⟨2/3,1/2⟩ b= ⟨8,6⟩

Determine m such that the two vectors are orthogonal.

24. 3i – 2j, 4i + 5mj
25. 9i – 16mj, i + 4mj

I can find the angle between vectors and find the dot product of two vectors.

Given that a= ⟨2,-3⟩,b= ⟨3,4⟩ and c= ⟨-1,5⟩, find the number,

26. a) a ·(b+c) b) a·b+a ·c
27. a) (2a+b) ·(3c) b) (a-b)·(b+c)

If c represents a constant force, find the work done if the point of application of c moves along the line segment from P to Q.

28. c=3i+4j; P(0,0), Q(5,-2)
29. c=6i+4j; P(2,-1), Q(4,3)
30. A constant force of magnitude 4 has the same direction as j. Find the work done if its point of application moves from P(0,0) to Q(8, 3).

31. A child pulls a wagon along level ground by exerting a force of 20 pounds on a handle that makes an angle of 30° with the horizontal, as shown in the figure. Find the work done in pulling the wagon 100 feet.

Problems from Troy Faulkner Pre-Calculus course.

Accuracy/Precision and Significant Figures

Lesson Objective: This lesson provides an understanding of accuracy, precision and significant figures. The topic is usually encountered within the first chapter and first days of a general chemistry course. Understanding this is important particularly in regards to any laboratory experience. Although there are numerous explanations provided in textbooks, they often use overly abstract analogies. The analogies often leave the students as much in the dark just as much as the central problem did.

Days of Instruction: 8 Days Total - (2 for accuracy/precision and 5 for significant figures). This may seem like an extreme number of days to some, however, a firm understanding of this topic is vital for any real job in the scientific field and all subsequent chemistry and physics learning. Therefore, we have chosen to hit this topic more than a typical textbook may.

Day 1:Pre-Test, Launch and Explore
Day 2:Hands on Significant Figures Activity
Day 3: Measuring lab - Measure length or masses with various precision instruments.
Have students try to add/subtract values
Day 4: Addition and subtraction numbers with significant figures
Day 5: Multiple and divide numbers with significant figures
Day 6: lab
Day 7: lab
Day 8: Review all concepts then Post Test

Minnesota State Standard(s):

Math	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	8.1.1.5	Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved.
Chemistry	The Nature of Science and Engineering	Physical and mathematical models are used to describe physical systems.	9C.1.3.4.1	Use significant figures and an understanding of accuracy and precision in scientific measurements to determine and express the uncertainty of a result.
Physics	The Nature of Science and Engineering	Physical and mathematical models are used to describe physical systems.	9P.1.3.4.1	Use significant figures and an understanding of accuracy and precision in scientific measurements to determine and express the uncertainty of a result.

Launch: Use the "Bridge" formula to demonstrate the importance of significant figures.

https://www.youtube.com/watch?v=Mipr1aAbLZ0

- How much weight can the bridge hold? Equation: $10,000(99 - (70\sqrt{2}))$

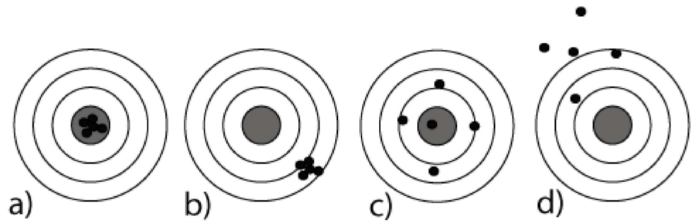
Explore: Assign different groups a different precision for the value of $\sqrt{2}$ (1, 1.4, 1.41, 1.414, etc). Give them time to calculate an answer. Use these widely varying answers as a discussion point to begin to show why the concepts of accuracy and precision are often important to an engineer while performing calculations.

Share: Have students write their work out on the board so that everyone can see each other's work. This should immediately start a conversation when students see how drastically different their answers are from each other even though they calculated the "same" thing.

Summarize: Summarize this activity by explaining that the concepts of accuracy and precision are very important in scientific fields (structural engineering, space travel, automobile engine design).

Accuracy & Precision Pre/Post Test

1. Below each of the following pictures, state whether it displays accuracy, precision, both, not neither.



____

2. Wendy is using a poorly calibrated electronic balance to measure the mass of a crucible. Her technique in making the measurement is very careful and she repeats it three times. What will likely characterize her measurements?

a) high accuracy and high precision
b) high accuracy and low precision
c) low accuracy and high precision
d) low accuracy and low precision

Three students have made multiple measurements of the solubility of sodium chloride (in g of NaCl per 100 g of water). Their results are summarized in the table below. The correct value for the solubility of sodium chloride is 35.9 g.

	Trial 1	**Trial 2**	**Trial 3**
Nicole	35.4 g	36.1 g	35.7 g
Matthew	31.8 g	34.1 g	41.5 g
Leonard	39.2 g	39.3 g	38.9 g

Refer to the above experiment to answer questions 3-5. Three students have made multiple measurements of the solubility of sodium chloride (in g of NaCl per 100 g of water). Their results are summarized in the table below. The correct value for the solubility of sodium chloride is 35.9 g.

3. Which student's measurements have high precision but low accuracy?

a) Nicole
b) Matthew
c) Leonard

4. Which student's measurements are both accurate and precise?

a) Nicole
b) Matthew
c) Leonard

5. Calculate the average of Matthew's three measurements. Comment on his accuracy and precision.

Significant Digits Pre/Post Test

1. 0.0507 has how many significant digits?

   a. 1 b. 2 c. 3 d. 4 e. 5

2. 2000 has how many significant digits?

   a. 1 b. 2 c. 3 d. 4

3. The number 10.10 has how many significant digits?

   a. 1 b. 2 c. 3 d. 4

4. 5.300 / 3.1 =

   a. 1.7 b. 1.71 c. 1.710 d. 1.7097 e. 2

5. 11.1 x 0.8 =

   a. 8. b. 8.8 c. 8.88 d. 8.9 e. 9.

Hands on Significant Figures Activity

The following activity was found on the Collinsville High School website. It will take a full class day, possibly 2 depending on level of the students and in how much depth the teacher chooses to go. It involves students creating their own custom rulers of various accuracy and taking measurements.

http://www.collinsville.k12.ok.us/webpages/documents/HS-FrameR/Chemistry/Lab_Significant%20Figures.pdf

Lab Activity
Significant Figures and Measurement

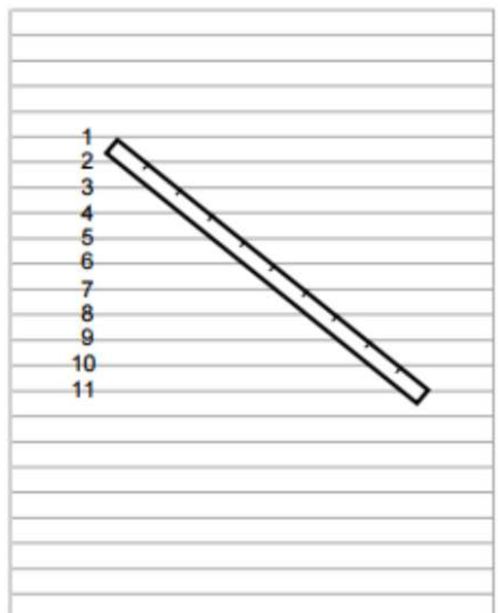
*Name*___________________________

*Date*____________

In this activity you will learn the concept behind significant figures and how to make measurements and calculations using that concept.

Materials: 1 wooden splint, 1 sheet of lined paper

Any measuring device is limited in its precision. To a large degree, the precision of a measurement is determined by the nature of measuring instrument itself. Specifically, to what degree the instrument is subdivided will determine to what decimal place the measurement will be reported. In science, we typically limit ourselves to measuring scales that have been divided based on powers of ten. A meter stick, for instance, might be divided into tenths, hundredths, and thousands of a meter if the smallest scale division marked on the ruler is the millimeter. Each of the digits you report in your measurement is considered a *significant figure*. In general, we report measurements by including all of the digits of which we are certain plus one estimated digit. In making measurements with a metric scale, it is conventional to report measurements to the smallest scale division marked on the scale, plus one estimate beyond the smallest scale division. There are exceptions to these rules that differ based on what you are measuring or the measurement technique, but these rules are generally followed.



Procedure:
For this activity, you will be measuring the geometric shapes on the back of this page using the stick that has been provided to you. All Trial 1 measurements are to be made with the side of the stick which has not been subdivided. This is the "uncalibrated" side of your stick. Trial 2 measurements will be made with the "calibrated" side of the stick.

1. Measure the following quantities using the unmarked stick.
   a. The length and width of the rectangle
   b. The base, height (altitude), and all three sides of the triangle
   c. The radius and diameter of the circle.

   Record your measurement in the trial 1 section of the data table, to the correct number of significant figures based on the concepts described above.

2. Make the following calculations based on the measurements.
   a. Perimeter and area of the rectangle
   b. Perimeter and area of the triangle
   c. Circumference and area of the circle

   Below the data table, show all work for each calculation. This should include starting with the equation written symbolically. For example, the area of a rectangle is equal to the length of the rectangle times the width of the rectangle. This should be written as $A = L \times W$. The next step in the calculation should show substitution of measurements (including units) into the equation. The final step should be writing the final answer with the correct units. Record the results of these calculations in the appropriate boxes in the trial 1 section of the data table.

Significant Figures and Measurement Activity *©Rex P. Rice, 2007* *Page 1*

Accuracy & Precision Practice

Name: Date: Pd:

Mark each set of numbers as having a high or low accuracy and precision.

	Accuracy High	Accuracy Low	Precision High	Precision Low
Ex: Object measured is 1.0 meter long 1.15 1.10 1.12 1.30	______	__X__	__X__	______
	average is high of 1.0		group is small	
1) Object measured is 50 cm length 52 60 48 41	______	______	______	______
2) Object measured is 27 mL volume 27.5 33.0 21.8 22.8	______	______	______	______
3) Object measured is 15 cm2 area 13.21 13.25 13.19 13.22	______	______	______	______
4) Object measured is 32 g mass 40 55 32 50	______	______	______	______
5) Object measured is 0.31 g/cm3 density 0.30 0.32 0.31 0.31	______	______	______	______
6) Object measured is 30C temperature 30.6 30.9 30.7 30.8	______	______	______	______

How to use labware activity

Below is a link to a practical application of accuracy, precision and significant figures in high school laboratory setting. It is a lab write up that will take most teachers 2 days to complete. It may take more or less depending on how deep you choose to cover it. The original has not been included in this document because it is a 9 page lab activity (see link below). It thoroughly covers how these topics apply to the use of different glass labware in the science setting.

http://web.gccaz.edu/~rob2108739/Density%20of%20Water/Accuracy%20and%20Precision%20of%20Glassware%20Fall%202013.pdf

Accuracy and Precision of Laboratory Glassware: Determining the Density of Water

During the semester in the general chemistry lab, you will come into contact with various pieces of laboratory glassware.

- beakers
- Erlenmeyer Flasks
- volumetric flasks
- pipettes
- burets
- graduated cylinders

Some of these pieces (e.g. beakers, Erlenmeyer Flasks) are used primarily to hold liquids during experiments. Upon closer inspection, you will also notice that they have graduations on the side to measure volumes. All of the glassware listed above can measure volumes. Why do we have so many pieces of glassware if they all do the same basic job of measuring volumes?

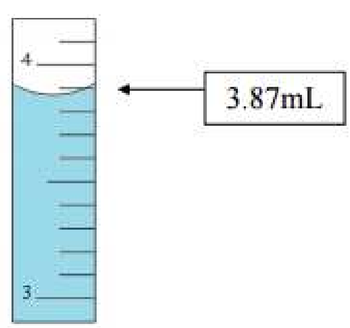
Laboratory glassware can generally be divided into two main types based on how they measure volumes:

- those that are manufactured to *contain* certain volumes
- those that are manufactured to *deliver* certain volumes

One of the first things that needs to be discussed before we can use any measuring device in the lab is something called significant figures. The first thing to realize is that there is no such thing as a perfect measurement. Even when using expensive lab equipment there some degree of uncertainty in measurement.

The general rule of thumb is: you can estimate one more digit past the smallest division on the measuring device. If you look at a 10mL graduated cylinder, for example, the smallest graduation is tenth of a milliliter (0.1mL). That means when you read the volume, you can estimate to the hundredths place (0.01mL).

Use the bottom of the meniscus to determine the volume in the 10mL graduated cylinder. Since the smallest division (graduation) is a tenth of a milliliter, we can estimate to a hundredth of a milliliter (0.01).



Portion of a 10mL graduated cylinder

Scientific Notation & Metric Prefixes

Lesson Objective: Students will be able to manipulate and interpret very large numbers and very small numbers expressed in the "exponent" format of scientific notation. Students will also be able to understand how to use metric prefixes (centi, milli, kilo, mega, etc) as a shorthand for commonly used scientific notations.

Days of Instruction:

Day 1: Pre-Test, Launch and Explore

Day 2: Have students work in small groups then share their results on the whiteboard using the problems in the problem set, selecting one problem from each section

Day 3: Using Prefixes to Describe the Size of Units of Storage in Computing, Post-Test

Minnesota State Standard(s):

Math	Geometry & Measurement	Calculate measurements of plane and solid geometric figures; know that physical measurements depend on the choice of a unit and that they are approximations.	9.3.1.3	Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.
Math	Number & Operation	Read, write, compare, classify and represent real numbers, and use them to solve problems in various contexts.	8.1.1.5	Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved.

**There is no science standard regarding scientific notation. However, it is still an important mathematical concept to understand so we have chosen to cover it.

Launch: Show the “Powers of Ten” video. https://www.youtube.com/watch?v=0fKBhvDjuy0 This is an old video, but very effective and showing the importance of understanding exponent notation while studying science. It would be advisable to write out the numbers used in the video on the whiteboard before class starts so students can see how big and small they really are.

Explore: After introducing the topic allow students to explore the website below. The website is an interactive flash animation similar to the video which will allow them to go back and forth at their leisure. Task them with finding 2 or 3 things that they find interesting.

http://htwins.net/scale2/

Share: Students should each present the specific things they found interesting in the animation. The instructor should then challenge their understanding of why it would be important for a scientist to be able to use scientific notation in the context of the ongoing discussion.

Summarize: The teacher will summarize the main points of the lesson of adding, subtracting, multiplying and dividing numbers in scientific notation and how to convert numbers within the metric system.

Scientific Notation Pre/Post Test

1. 0.0057 is properly shown in scientific notation as

   a. 57×10^{4}
   b. 57×10^{-4}
   c. 5.7×10^{3}
   d. 5.7×10^{-3}
   e. 5.7×10^{-4}

2. 500.7 is properly shown in scientific notation as

   a. 5007
   b. 5007×10^{-1}
   c. 5.007
   d. 5.007×10^{2}
   e. 5.007×10^{-2}

3. 2.3×10^{4} is the same as

   a. 0.000023
   b. 0.00023
   c. 2300
   d. 23,000
   e. 230,000

4. 3.2×10^{-1} is the same as

   a. 0.032
   b. 0.32
   c. 3.2
   d. 32
   e. 320

5. $(7.1 \times 10^{7}) \times (1.2 \times 10^{-5}) =$

   a. 8.5×10^{12}
   b. 8.5×10^{2}
   c. 8.5×10^{-2}
   d. 8.5×10^{-12}
   e. 8.5×10^{-35}

Scientific Notation Pre/Post Test continued...

6. $(5.0 \times 10^{7}) / (2.0 \times 10^{10}) =$

   a. 2.5×10^{-10}
   b. 2.5×10^{-4}
   c. 2.5×10^{-3}
   d. 2.5×10^{4}
   e. 2.5×10^{10}
   f. 2.5×10^{21}

Convert each of the following based on the metric prefixes used shown.

7. 32 cm = ________________ mm

8. 765 ms = _______________ s

9. 3.6 km = _______________ cm

10. 15 L = _______________mL

Numbers Practice!!!

The below problems are done in small groups and groups share on the whiteboard.

Convert each of the following to the required units.

1.) 32 cm = ________________ mm

2.) 765 ms = _______________ s

3.) 3.6 km = _______________ cm

4.) 15 L = _______________mL

5.) 45 dL = _______________ L

6.) 8000 mg = _______________ g

7.) 67 kg = _______________ Mg

8.) 0.024 g = _______________ micrograms

9.) 14 cmol = _______________ nmol

Simplify:

10.) $(2 \times 10^{4}) \times (8 \times 10^{4}) =$ __________________

11.) $(6 \times 10^{2}) / (11 \times 10^{8}) =$ __________________

12.) $(3.2 \times 10^{8}) \times (4.6 \times 10^{2}) =$ __________________

13.) $(-2.98 \times 10^{7}) \times (0.56 \times 10^{-5}) =$ __________________

14.) $(7.8 \times 10^{-3}) / (8 \times 10^{2}) =$ __________________

15.) $(6.54 \times 10^{3}) / (-4.78 \times 10^{9}) =$ __________________

16.) $(2 \times 10^{-3}) \times (1.45 \times 10^{-2}) =$ __________________

17.) $(2.65 \times 10^{4}) / (3.79 \times 10^{-6}) =$ __________________

Put the following numbers into scientific notation:

18.) 3,000,000,000 m/s = ________________________

19.) 1,400,000 km = ________________________

20.) 0.0089 sec = ________________________

21.) 36,000 sec = ________________________

22.) 0.00000000976 = ________________________

23.) 4980 g = ________________________

24.) .045 mol = ________________________

Write the following numbers out long hand:

25.) 3.00×10^{8} = __________________________

26.) 2.13×10^{-4} = __________________________

27.) 6.8×10^{3} = __________________________

28.) 9.34×10^{-6} = __________________________

Give the number of significant figures:

29.) 173.2 = ______

30.) 205 = ______

31.) 4000 = ______

32.) 0.025 = ______

33.) 700 = ______

34.) 0.09050 = ______

Using Prefixes to Describe the Size of Units of Storage in Computing

The following website has an excellent way to relate metric prefixes to a real world use via computer technology. Students should already have an intuition about many of the topics covered but by the end should understand why we have terms such as “megabye” and “gigabyte” in the computer data world.

https://mathspace.co/learn/world-of-maths/focus-study-maths-and-communication/data-18820/using-prefixes-to-describe-the-size-of-units-of-storage-23/